**2023 Academic Challenge**

**Regional Mathematics Exam**

1. A board game player is trying to roll doubles with two, six-sided dice. Given that the player’s roll is at least a 9, what is the probability of getting doubles on that roll?

a) 1/2 b) 1/6 c) 1/5 d) 1/3 e) 2/3

**Conditional probability. We are given that the roll is at least a 9, and the probability of that happening is 4/36 for a 9, 3/36 for a 10, 2/36 for an 11, and 1/36 for a 12. Those probabilities add up to 10/36. A 10 and a 12 would be doubles. So the probability of doubles given at least a 9 is 2/10, or 1/5. Answer is C.**

2. Which trinomial has two distinct, integral roots that are both prime numbers?

a) $x^{2}$ + 7$x$ + 8 b) 2$x^{2}$ - 16$x$ + 30 c) 3$x^{2}$ + 6$x$ - 9 d) 2$x^{2}$ + 6$x$ + 10 e) $x^{2}$ - 8$x$ + 16

**The answer is B. Once a GCF of 2 is factored out, the resulting trinomial is x^2 - 8x + 15. This factors into (x-5)(x-3), which has roots of 5 and 3.**

3. If a circle is created by the equation $x^{2}$ + $y^{2}$ = 64, and another circle is centered at (2,2) and has a diameter of 8 units, what is the area of the region between the two circles?

a) 48π square units b) 192π square units c) 16 square units d) 24π square units

e) 16π square units

**The larger circle has an area of 64pi, and the smaller circle, which is fully enclosed in the first circle, has an area of 16pi. 64pi - 16 pi equals 48pi square units. The answer is A.**

4. Which type of polygon cannot be inscribed in a circle? Assume the scaling of the polygon and the scaling of the circle are similar.

a) isosceles trapezoid b) right triangle c) scalene triangle

d) quadrilateral with angles of 30, 45, 120, and 165e)rhombus with right angles

**The opposite angles of an inscribed quadrilateral must add up to 180 degrees, and triangles can always be inscribed. All of the choices could be inscribed except for Answer D.**

5. In the set of imaginary numbers, $i$ = $\sqrt{-1}$. What is the value of $i^{347}$?

a) 1 b) -1 c) $i$ d) -$i$ e) 0

**i^4 = 1. The closest multiple of 4 less than 347 is 344. So i^347 = i^344 \* i^3 = i^3 = -i. The answer is D.**

6. A square garden has an area of 625 square feet. If square tiles, each with an area of 1 square foot, are to be placed around the edges of the garden, how many tiles are needed?

a) 625 b) 125 c) 121 d) 100 e) 96

**Each corner tile completes two sides. So although each side of the square is 25 feet, and the perimeter is 100 feet, only 96 tiles are needed. The answer is E.**

7. If the height of a rectangle is 5 inches, and the diagonal is 13 inches, what is the area of the rectangle?

a) 65 square inches b) 40 square inches c) 60 square inchesd)32.5 square inches e) 45 square inches

**The height and the diagonal must form two sides of a right triangle. Using the Pythagorean Theorem, the base of the rectangle must be 12 inches. 5 \* 12 = 60 square inches for Answer C.**

8. In a system with the equations 2$x$ + 3$y$ = 12 and $x$ + 4$y$ = 8, what ordered pair represents the intersection of the lines?

 a) (0.8, 4.8) b) (0.6, 4.4) c) (4.2, 0.6) d) (4.8, 0.8) e) The lines do not intersect.

**Using either elimination or substitution, the coordinate pair of the intersection would be Answer D.**

9. In the conic section generated by the equation ($y^{2}$/9) + ($x^{2}$/16) = 1, what is the length of the major axis?

a) 8 units b) 6 units c) 4 units d) 3 units e) 10 units

**The square root of the larger denominator is half the length of the major axis. 4 \* 2 = 8, so the answer is A.**

10. A sphere has a diameter of 12 inches. What is the volume of the sphere, to the nearest integer?
(V = 4/3(π)$r^{3}$)

a) 402 cubic inches b) 15875 cubic inches c) 905 cubic inches

d) 8930 cubic inches e) 201 cubic inches

**The radius would be 6 inches. 6^3 = 216. Multiplying 216 by 4/3 and pi gives Answer C.**

11. In the quadratic function $x^{2}$ + 3$x$ + 6, what is the slope of the function at $x$ = 1?

1. 3 b) 5 c) 6 d) 10 e) 1

**The derivative of the function is 2x + 3. When x = 1, the derivative output is 5. Answer B.**

12. What are the discontinuities of the function ($x^{2}$ - 2$x$ - 15)/($x^{2}$ - 4$x$ - 5)?

a) Vertical asymptote at $x$ = 5, vertical asymptote at $x$ = -1.

b) Horizontal asymptote at $y$ = 0, vertical asymptote at $x$ = 5, vertical asymptote at $x$ = -1.

c) Horizontal asymptote at $y$ = 1, vertical asymptote at $x$ = 5, vertical asymptote at $x$ = -1.

d) Horizontal asymptote at $y$ = 0, hole at (5, 4/3).

e) Horizontal asymptote at $y$ = 1, vertical asymptote at $x$ = -1, hole at (5, 4/3).

**The numerator factors to (x-5)(x+3), and the denominator factors to (x-5)(x+1). Because the (x-5) factors cancel, there would be a hole at x=5. With this x value, the output of the original function is 4/3. With (x+1) still in the denominator, there would be an asymptote at x = -1. The leading degrees of both the numerator and the denominator are 2, so the horizontal asymptote would be y = 1. Answer E.**

13. A list of data is composed of the numbers 1, 4, 6, 8, 8, 12, 17. What number can be added to the list without changing either the mean or the median?

a) 0 b) 2 c) 6 d) 8 e) Not possible.

**In the original set, the mean is 8 and the median is 8. If an 8 is added to the middle, the mean and the median would stay the same. Answer D.**

14. What is the least common denominator of the fractions

6/($x^{2}$ -7$x$ – 18)

3/($x^{2}$ + 8$x$ + 12)

5/($x^{2}$ – 12$x$ + 27)

a) ($x$-9)($x$+2) b) ($x$+2)($x$+6)($x$-9) c) ($x$-9)($x$+2)($x$+6)($x$-3) d) ($x$+2)($x$+6)

e) ($x$+2)($x$+6)($x$-3)($x$+9)

**Fraction One has factors of (x-9) and (x+2). Fraction Two has factors of (x+2) and (x+6). Fraction Three has factors of (x-9) and (x-3). Because none of those factors is in all three denominators, the common denominator would consist of four factor pairs multiplied together. Answer C.**

15. In a right triangle, side AB is 8, side BC is 15, and side AC is 17. What is the tangent of angle A?

a) 8/17 b) 15/8 c) 17/8 d) 8/15 e) 17/15

**The side opposite to Angle A is BC, with a measure of 15. The side adjacent to Angle A is AB, with a measure of 8. Tangent is opposite/adjacent, which is 15/8. Answer B.**

16. What trigonometric value is undefined?

a) sin π/3

b) cos 2π/3

c) tan 3π/2

d) sec π

e) csc π/2

**The cosine value of 3pi/2 is zero. So the tangent at 3pi/2 is undefined. Answer C.**

17. If the side measure of a cube is doubled, what happens to the surface area and the volume of the cube?

a) Surface area increases by a factor of 2; volume increases by a factor of 8.

b) Surface area increases by a factor of 4; volume increases by a factor of 8.

c)Surface area increases by a factor of 4; volume increases by a factor of 6.

d) Surface area increases by a factor of 2; volume increases by a factor of 6.

e) Surface area increases by a factor of 8; volume increase by a factor of 16.

**If a side of a cube is doubled, the area increases by a factor of 2 squared, and the volume increases by a factor of 2 cubed. Answer B.**

18. If only integers greater than 1 are considered, which set of integers always has an odd number of distinct, integral factors?

a) Fibonacci numbers

b) prime numbers

c) odd numbers

d) perfect squares

e) even numbers

**Only perfect squares always have an odd number of distinct factors. These numbers have a set of distinct factor pairs, and the remaining factor pair is the square root times itself. The answer is D.**

19. What is the value of

$log\_{9}$3 – log 0.1?

a) -3/2 b) -1/2 c) 0 d) 1/2 e) 3/2

$log\_{9}$**3 = 1/2 because in exponential form, 9^(1/2) = 3. The log of 0.1 is -1 because 10^(-1) = 0.1. 1/2 - -1 = 1/2 + 1 = 3/2. Answer E.**

20. Which linear equation generates a line that passes through the highest points of the
function 1/2 (cos 3($x$+6)) + 4?

a) $y$ = 1 b) $y$ = 2 c) $y$ = 4.5 d) $y$ = 4 e) $y$ = 6

**The peak of the base cosine function is 1. Multiplying the peak by 1/2 and then adding 4 would put the new peak value at 4.5. The equation y = 4.5 would generate a horizontal line that would pass through the peaks. Answer C.**

21. A basketball player is shooting toward a regulation 10-foot basket. If the player is allowed to take steps back as needed, which equation would generate a shooting arc that would allow the ball to reach the basket?

a) $y$ = -$x^{2}$ + 2$x$ + 6 b) $y$ = -$x^{2}$ + 4$x$ + 6 c) $y$ = -$x^{2}$ + 4$x$ + 5 d) $y$ = -2$x^{2}$ + 8

e) $y$ = -2$x^{2}$ + 4$x$ + 6

**We need a parabola opening down that has a vertex point with y = 10. Answer B generates this vertex point.**

22. The Richter scale of the magnitude of an earthquake is measured using logarithmic comparisons. If an earthquake measures 5.2 on the Richter scale, what is the magnitude of an earthquake that is 100 times more powerful?

1. 5.40 b) 5.22 c) 5.90 d) 6.22 e) 7.20

**Log Earthquake 1 = 5.2. Log Earthquake 2 = Log ((Earthquake 1) \* 100) = Log Earthquake 1 + Log 100 = 5.2 + 2 = 7.2. The answer is E.**

23. Fahrenheit temperatures are converted to Celsius by subtracting 32 and then multiplying by 5/9. If a set of Fahrenheit temperatures has a standard deviation of 4.5, what would the standard deviation be after converting the set to Celsius?

a) -15.28 b) -27.5 c) -29.5 d) 2.5 e) 20.28

**Only multiplication and division change the value of a standard deviation. 4.5 \* (5.9) = 2.5. Answer D.**

24. What is the probability of drawing two hearts from a standard deck of 52 cards if both cards are drawn without replacement?

a) 1/169 b) 2/169 c) 1/16 d) 1/12 e) 1/17

**The probability of a heart on Draw 1 is 13/52, or 1/4. Without replacement, the probability of a heart on Draw 2 is 12/51, or 4/17. This is an AND probability, which requires multiplication. 1/4 \* 4/17 = 1/17. Answer E.**
25. A cone has a radius of 4.5 meters and a height of 18 meters. If water is added to the cone, and the water level reaches a height of 10 meters, what is the volume of the water in the cone at that point? The equation for volume of a cone is (1/3)π$r^{2}h$.

a) 381.70 cubic meters b) 65.45 cubic metersc)128.28 cubic meters d) 212.06 cubic meters

e) 94.25 cubic meters

**The height is 4 times the radius throughout the cone. So when the height is 10, the radius is 2.5. When you put this information into the volume equation, the answer is B.**

26. If $y$ varies inversely with $x$, and $y$ = 5 when $x$ = 3, calculate the value of $y$ when $x$ = 10.

a) 1/3 b) 2/3 c) 10/3 d) 3/2 e) 1/2

**For inverse variation, y = k/x, so k = xy. Then k = 15 from the original information. When x = 10, y = 15/10 = 3/2. The answer is D.**

27. If a car is traveling 60 miles per hour, what is its speed in feet per second? (There are 5,280 feet in a mile.)

a) 0.114 feet/second b) 1.47 feet/second c) 14.67 feet/second d) 88 feet/second

e) 112 feet/second

**Unit conversion: 60 miles/1 hour \* 1 hour/3600 seconds \* 5280 feet/1 mile = 88 feet/second. Answer is D.**

28. Determine the number of unique arrangements of the letters in the word LEVERAGE.

a) 40,320 b) 13,440 c) 6,720 d) 512 e) 64

**8!/3! = 8\*7\*6\*5\*4 = 6,720. The answer is C.**

29. Along the unit circle, with a domain of [0, 2π), how many locations have an equal value for tangent and cotangent?

a) 0 b) 1 c) 2 d) 4 e) 8

**We need sine and cosine to be the same magnitude and the same sign. This happens at pi/4, 3pi/4, 5pi/4, and 7pi/4. The answer is D.**

30. A city is losing 2 percent of its population each year. If it has 63,000 people in 2023, how many people will it have in 2038? Round to the nearest integer.

a) 44,100 b) 46,530 c) 50,400 d) 53,550 e) 55,250

**The equation for decay is Starting Amount \* (1- rate of decay) ^ time. So 63,000(0.98)^15 = 46,530.**